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INCREASES IN THE LOW ENERGY COSMIC RAY INTENSITY AT THE FRONT OF PROPAGATING INTERPLANETARY SHOCK WAVES

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INCREASES IN THE LOW ENERGY COSMIC RAY INTENSITY AT THE FRONT OF PROPAGATING INTERPLANETARY SHOCK WAVES

by

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ABSTRACT

A simple model is discussed which can account for many of the features observed in the pulse-like increases in the low energy cosmic ray intensity that occur at the front of propagating interplanetary shock waves. It is assumed that low energy particles are swept up by the shock, but because of extensive scattering by magnetic field irregularities remain near the shock front forming the pulse. It is found that the intensity can increase substantially at the shock as particles gain energy by making repeated collisions with the moving shock front. The behavior of particles which accumulate at the shock is illustrated with some solutions to the time-dependent Fokker-Planck equation which governs cosmic ray behavior allowing for convection, diffusion, and particle energy changes. The predicted pulse shapes agree reasonably well with the observations provided that the diffusion coefficient parallel to the magnetic field is $\sim 10^{19}$ cm.^{2} sec.$^{-1}$ for 1 MeV protons.

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I. INTRODUCTION

In recent papers, Ogilvie and Arens (1970), and Armstrong, et al. (1970) discuss a number of events in which the intensity of low energy particles (.3-10 MeV/nucleon) increases abruptly at the front of propagating interplanetary shock waves, forming a pulse-like structure. These increases typically begin some 60 minutes before the arrival of the shock, reaching a peak at the shock passage which is 5-50 times greater than the undisturbed intensity ahead of the shock. The half-widths of the peaks are $\sim$ 5-10 minutes in duration, and following the shock, the intensity in general drops abruptly (within a few minutes) to its undisturbed value. Increases similar to the ones described here have also been reported by Singer (1970).

Ogilvie and Arens (1970) discuss the possibility that these increases result when low energy particles are compressed between the oncoming interplanetary shock and the earth's bow shock in the manner discussed by Axford and Reid (1963). While we have no objection to this mechanism in principle, it should be noted that there is at least one event in which the mechanism does not appear to be the dominant cause of the increase. Armstrong, et al. (1970) observe a large event on 11 January 1968 from Explorer 35, which at the time was apparently located in the magnetosheath. As Armstrong et al. point out, this observation of a particle increase behind the bow shock, which is similar in form and in magnitude with increases observed in the interplanetary medium, is inconsistent with a picture in which the bow shock plays an important role in the particle acceleration. This event was also observed from Explorer 33 which was located some 50 earth radii in front of the bow shock (Armstrong et al., 1970) and from Explorer 34 which was located near the bow shock but in the interplanetary medium (Ogilvie and Arens, 1970). Although it is some-

what difficult to establish, the event appears to begin at Explorer 33 some 20 minutes before it begins at Explorer 34. This apparent time separation between the onsets at the two satellites is consistent with a picture in which the particle increase propagates with the shock front, and not with one in which intensity increases uniformly between the bow shock and the oncoming interplanetary shock.

In this paper we will consider another mechanism: that low energy particles are swept ahead of a propagating interplanetary shock, but are scattered extensively by magnetic field irregularities and are thus confined to remain near the shock front forming the pulse. We will find that the particle intensity can increase substantially at the shock as particles gain energy by making repeated collisions with the moving shock front. The width of the pulses can be used to estimate the magnitude of the particle diffusion coefficient at low energies, and indeed the predicted pulse shapes agree reasonably well with the observations provided that the diffusion coefficient parallel to the magnetic field is $\sim 10^{19}$ cm.^{2} sec.$^{-1}$ for 1 MeV protons.

In section II we will illustrate the behavior of particles reflecting off a propagating interplanetary shock with some solutions to the time-dependent Fokker-Planck equation which governs cosmic ray behavior allowing for convection, diffusion, and particle energy changes. We will show that these solutions provide reasonable fits to some observed events, and on the basis of these solutions we will infer some of the general properties expected for particle increases at shock fronts.

II. DISCUSSION

Consider a spherically symmetric model of the interplanetary medium in which cosmic ray particles behave diffusively as they are scattered

among magnetic irregularities moving radially outward with the solar wind. In such a model the cosmic ray number density U(r,T,t) and streaming S(r,T,t) (radial current density), per unit interval of kinetic energy T, satisfy the equations:

$$\frac{\partial U}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 S) = -\frac{V}{3}\frac{\partial^2(\alpha TU)}{\partial r\,\partial T} \tag{1}$$

and

$$S = VU - \kappa\frac{\partial U}{\partial r} - \frac{V}{3}\frac{\partial}{\partial T}(\alpha TU) \tag{2}$$

Here, $\kappa(r,T)$ is the particle diffusion coefficient, V(r) is the solar wind speed, and $\alpha(T) = (T + 2T_0)/T + T_0)$, with T_0 the rest energy of a particle (Gleeson and Axford, 1967).

On eliminating S between equations (1) and (2), a Fokker-Planck equation is obtained for U:

$$\frac{\partial U}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}(r^2\ VU) - \frac{1}{3r^2}\frac{\partial}{\partial r}(r^2 V)\frac{\partial}{\partial T}(\alpha TU) = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2\kappa\frac{\partial U}{\partial r}) \tag{3}$$

This equation contains the usual terms that describe the convection and diffusion of cosmic rays, together with an additional term (the third term on the left side), which represents the effect of the energy changes resulting from the expansion of the solar wind (Parker, 1965).

Increases in the particle intensity accompanying shocks are in general always observed on the upstream side of the shock. The enhanced intensity drops abruptly (by a factor of 5 -50 within a few minutes) at the shock passage (Ogilvie and Arens, 1970; Armstrong, et al., 1970). These observations indicate that low energy particles ($\sim$ 1 MeV/nucleon) approaching

the shock from the upstream side will be effectively reflected at the shock front or by magnetic irregularities directly behind the shock. These particles will then be swept up by the moving shock and can accumulate in a pulse-like structure at the shock front provided that interplanetary conditions ahead of the shock are suitable.

If the magnetic irregularities behind the shock are responsible for the particle reflection, it is probably reasonable to assume that the particles do not make numerous small angle collisions in reversing their directions, but rather must reflect directly, in 'one collision', off field variations which are comparable in size with the particle gyroradius (assuming that such large variations exist). Behind the shock, the interplanetary plasma, and hence the magnetic field irregularities, move at a speed somewhat slower than the shock speed. If particles undergo numerous collisions behind the shock, then relative to the shock front they will be effectively convected in a direction opposite to that of the shock propagation, i.e. the shock will leave the particles behind. If such were the case, we would expect that the intensity behind the shock was not significantly smaller than intensity ahead, contrary to what is observed. For simplicity, we assume that all the particles are reflected at the shock front by some reflecting medium moving with speed V', e.g. V' could be the shock speed or the speed of the irregularities behind the shock. It can be shown that the streaming at the shock is then:

$$S = V'U - \frac{V'}{3}\frac{\partial}{\partial T}(\alpha T U) \qquad (4)$$

The second term on the right side of this equation describes the effect of the energy gains suffered by the particles upon collision with the shock.

We can illustrate the behavior of cosmic ray particles ahead of the shock with a similarity solution to the Fokker-Planck equation (equation (3)). We assume that the diffusion coefficient is given by $\kappa = \kappa_0 r$, where κ_0 is a constant (independent of T), that the solar wind speed V is constant, and that initially ($t \to 0$) the cosmic ray number density U_i is given by:

$$U_i\,(r,T) = AT^{-\mu}\, r^{\lambda} \tag{5}$$

where $\lambda = V/2\kappa_0 - 1 - \sqrt{[(1 + V/2\kappa_0)^2 + 4V(\mu - 1)/3\kappa_0]}$. U_i is a solution to the steady-state form of equation (3) with the above forms for κ and V, and with $\alpha = 2$ (e.g. U_i describes the behavior of a steady flux of solar cosmic rays). We will be concerned here only with low energy particles ($T << T_0$), and hence we have taken α to be constant and equal to 2. On assuming that the streaming at the shock is given by equation (4), and that $U \to U_i$ as $r \to \infty$, we show in Appendix A that equation (3) is satisfied by a similarity solution in terms of the variable $\eta = r/V_s t$, with V_s the speed of the shock (V_s is taken to be constant):

$$U(T,\eta) = A'T^{-\mu} r^{\lambda}\, \Gamma(a, V_s\eta/\kappa_0) + AT^{-\mu} r^{\lambda}\ , \quad \text{for } \eta = r/V_s t \geq 1 \tag{6}$$

where $A' = \dfrac{[(V'-V)(1+2\mu)/3 + \kappa_0\lambda]A}{[\kappa_0(V_s/\kappa_0)^a \exp(-V_s/\kappa_0) - ((V'-V)(1+2\mu)/3 + \kappa_0\lambda)\Gamma(a, V_s/\kappa_0)]}$

and $a = V/\kappa_0 - 2 - 2\lambda$. $\Gamma(a, z) = \int_z^{\infty} e^{-w} w^{a-1}\, dw$ is the incomplete gamma function. The particle intensity J is determined from the number density by $J = vU/4\pi$, where v is particle speed.

The first term on the right side of equation (6) describes the behavior of particles which are swept up by the shock provided, of course, that A' is positive. From equation (2) we see that the streaming in the interplanetary medium corresponding to the initial number density U_i (equation (5)) is

$$S = [V(1 + 2\mu)/3 - \kappa_0\lambda]U_i \quad (7)$$

Hence, for the numerator of A' to be positive, the shock must impart a streaming to the particles of the initial distribution ($S = V'(1+2\mu)U_i/3$; see equation (4)) which exceeds the streaming these particles would normally have in the interplanetary medium (equation (7)). We anticipate that particles with energies $\lesssim$ 10 MeV/nucleon (which are presumably of solar origin (Kinsey, 1970)) will undergo extensive scattering in the interplanetary medium and will therefore have a sufficiently small outward streaming to be 'overtaken' by the shock. These particles will then accumulate at the shock front, unlike particles with higher energies, which have too large a streaming, or which do not reflect off the shock. Further, when the diffusion coefficient is small, particles which accumulate at the shock will 'bounce' back and forth between the shock front and field irregularities ahead of the shock.* On each 'complete bounce' (back and forth) a particle's speed is increased roughly in proportion to (V' - V).

*We have assumed throughout this discussion that low energy particles can be backscattered, scattered through $\sim 180^o$, within a short distance in the interplanetary medium, i.e. that a particle which is reflected off the shock is likely to be scattered through a large angle by field irregularities ahead of the shock, and return to encounter the shock again.

Accordingly, at a given energy, the number density at the shock is increased provided $\mu > 1$. Requiring that the denominator of A' is positive is equivalent to requiring that the streaming which results from the particle energy gains at the shock front does not exceed the rate at which these particles are transported away from the shock, i.e.

$$\frac{(V - V')}{3} \frac{\partial}{\partial T} (\alpha T U') < (V - V')U' - \kappa \frac{\partial U'}{\partial r} \tag{8}$$

where U' is the number density of particles swept up by the shock (as is determined by the similarity solution), evaluated at the shock front ($\eta = 1$). If the condition given in (8) is violated, particles will accumulate at the shock front, the number density increasing markedly in time due to the repeated accelerations in violation of our assumption that the solution is similar.

In the larger events which have been observed (where the intensity increases by, say, a factor of $\sim$ 25 - 50) the number of particles at the shock front presumably does increase markedly in time, tempered of course by the rate at which particles leak through the shock front, or are lost through other means. We therefore anticipate that the solution given in equation (6) can be used to describe only the smaller events, e.g. where the intensity increases by a factor $\sim$ 5. Clearly, the intensity increases described by equation (6) are primarily density increases, whereas in the larger events (which we will discuss later) the intensity increases result primarily from particle accelerations.

In the limit when the particles undergo extensive scattering ahead of the shock (i.e. when V/κ_o and $V_S/\kappa_o >> 1$), the solution given in equation (6) has the asymptotic form (Erdélyi, et al., 1953):

$$U \sim AT^{-\mu} r^{\lambda} \left[\frac{(V'-V)(1+2\mu)/3}{(V_s-V)-(V'-V)(1+2\mu)/3} \exp\left(-\frac{V_s}{\kappa_o}(\eta-1)\right)\eta^{V/\kappa_o}\left(\frac{V_s\eta-V}{V_s-V}\right)+1\right] \quad (9)$$

In this limit $\lambda \simeq -2(1+2\mu)/3$. Note that at a given value of r the shape of the pulse is independent of λ, and hence independent of our choice for the initial distribution. It can be shown that the gradient of the term in equation (9) which describes the bahavior of the particles swept up by the shock (the first term on the right side) is

$$\frac{1}{U'}\frac{\partial U'}{\partial r} \sim \frac{V - V_s}{\kappa_o r} \quad (10)$$

when evaluated at the shock front ($\eta = 1$). The half-width of the particle increases (in spatial extent) should then be roughly $\kappa/(V_s-V)$. Indeed, this result should hold generally (not just for the special form of the diffusion coefficient assumed here) provided of course that the intensity at the shock is not increasing too rapidly in time. It should be possible to use this expression for the half-width to estimate the local diffusion coefficient at low energies, although, as we shall discuss later, care must be taken to distinguish between the diffusion parallel to the shock normal and diffusion along the field lines.

In figure 1 we have plotted the relative intensity increase determined by the solution given in equation (6), using the values for κ_o, V', and μ which give the best fit to the data obtained by McDonald (unpublished), and by Williams and Arens (Ogilvie and Arens, 1970) for the event observed on 29 November 1967. The particles observed in this event are presumably mainly protons with energies $\sim$ 1 MeV. We find that $\kappa = \kappa_o r = .656 \times 10^{18}$ cm.^{2} sec.$^{-1}$ at r = 1 A.U., V' = 433 km. sec.$^{-1}$, and $\mu = 3$. The solar wind speed was assumed to be 350 km. sec.$^{-1}$ in agreement with the observed solar wind speed ahead of the shock, and the shock speed was taken to be 600 km. sec.$^{-1}$. The interplanetary plasma immediately following the shock on 29 November had a

speed $\sim$ 450 km. sec.$^{-1}$ (Ogilvie and Arens, 1970), and hence we have assumed that it is magnetic irregularities embedded in this plasma which reflect the particles at the shock front.

To describe the larger events, where the intensity increases are presumably the result primarily of particle accelerations, we consider the somewhat simpler problem in which the particle behavior ahead of the shock is governed by a simple convection-diffusion equation:

$$\frac{\partial U}{\partial t} + \frac{\partial}{\partial r}(VU) = \frac{\partial}{\partial r}\left(\kappa \frac{\partial U}{\partial r}\right) \tag{11}$$

Equation 11 differes from equation (3) only in that here we have assumed that the geometry of the interplanetary medium is planar rather than spherically symmetric. The effects of spherical symmetry on the particle pulse, which after all has a limited spatial extent, should be small.

We take κ and V to be constants, and assume that initially ($t \to 0$) the cosmic ray number density is given by $U_i = AT^{-\mu}$ (independent of r). On assuming that the streaming at the shock is given by equation (4) (with $\alpha = 2$), and that $U \to U_i$ as $r \to \infty$, we show in Apprndix B that equation (11), written in terms of the variables t and $\zeta = r - V_s T$ (V_s is the constant speed of the shock), is satisfied by:

$$\begin{aligned} U(T,\zeta,t) = AT^{-\mu}\Bigg[& \frac{V_2 C}{2(V_1 - CV_2)} \exp\left(-\frac{V_1\zeta}{\kappa}\right) \operatorname{erfc}\left(\frac{\zeta}{2\sqrt{\kappa t}} - \frac{V_1}{2}\sqrt{\frac{t}{\kappa}}\right) \\ & - \frac{1}{2}\operatorname{erfc}\left(\frac{\zeta}{2\sqrt{\kappa t}} + \frac{V_1}{2}\sqrt{\frac{t}{\kappa}}\right) \\ & + \frac{(V_1/2 - CV_2)}{(V_1 - CV_2)} \exp\left(-\frac{CV_2\zeta}{\kappa} + \frac{CV_2^2}{\kappa}\left(C - \frac{V_1}{V_2}\right)t\right) \operatorname{erfc}\left(\frac{\zeta}{2\sqrt{\kappa t}} + \left(\frac{V_1}{2} - CV_2\right)\sqrt{\frac{t}{\kappa}}\right)\Bigg] \end{aligned} \tag{12}$$

where $V_1 = V_s - V$, $V_2 = V' - V$, and $C = (1 - 2\mu)/3$. $\operatorname{erfc}(z) = \frac{2}{\sqrt{\pi}} \int_z^\infty e^{-w^2} dw$ is the complementary error function. It should be noted that with the above choice for the initial number density, U_i, particles will always be swept

up by the shock regardless of the choice for the particle diffusion coefficient. The streaming in the interplanetary medium corresponding to U_i ($S = V(1 + 2\mu)U_i/3$) is always less than the streaming which the shock imparts to these particles ($S = V'(1 + 2\mu)U_i/3$). Realistically, then, the solution given in equation (12) should be used only to describe the behavior of low energy particles which, since they undergo extensive scattering, are in fact 'overtaken' by the shock.

The solution given in equation (12) has a fundamentally different behavior depending on whether V_1 is greater than or less than CV_2. If V_1 is greater than CV_2, then for large times only the first term on the right side is needed to describe the particle increase and the solution tends to

$$U(T,\zeta,t) \sim AT^{-\mu} \left[\frac{V_2 C}{(V_1 - CV_2)} \exp\left(- \frac{V_1 \zeta}{\kappa} \right) + 1 \right] \text{as } t \to \infty \tag{13}$$

This asymptotic form is essentially the same as the similarity solution (cf equation (9)) both in the magnitude of the particle increase at the shock front ($\zeta = 0$), and in the half-width of the peak, which is again (in spatial extent) $\sim \kappa/(V_s - V)$. However, if V_1 is less than CV_2, then the third term in equation (12) grows exponentially in time and will be the dominant term in describing the particle increase. Here the spectrum has a sufficiently steep negative slope ($(1 + 2\mu)/3 > (V_s - V)/(V' - V)$) so that the particles accumulate at the shock front due to accelerations at a rate faster than they are transported away from the shock (It can be seen from equation (8) that the particle increase cannot be described by a similarlity solution when $V_1 < CV_2$ by noting that in this case $\partial U'/\partial r \sim -V_1 U'/\kappa$.) Indeed, Ogilvie and Arens (1970) find that spectra

with steep negative slopes are characteristic of the larger particle increases observed.

Of course the intensity at the shock does not increase indefinitely in time. For example, it might be possible that particles are accelerated by the shock only over a short radial distance in the vicinity of the orbit of earth. Particles may undergo less scattering and hence less acceleration near the sun than they do at earth (Fan, et al., 1968), and also, beyond the orbit of earth the acceleration should be diminished since here it is reasonable to expect that the shock speed will be considerably reduced. Note also that nowhere in this treatment have we allowed for the fact that some fraction of the particles will not be reflected at the shock front, but rather will leak into the region behind the shock and be lost to the acceleration process. Clearly, the rate at which particles are lost by this or other means will significantly effect the rate at which particles accumulate at the shock front. Indeed, it is possible that a balance is achieved between accumulation and loss and the intensity at the shock remains fairly constant.

In figure 2 we have plotted the relative intensity increase determined by the solution given in equation (12), using the values for V', μ, and κ which give the best fit to the data obtained by McDonald (unpublished) and by Williams and Arens (unpublished) for the event observed on 11 January 1968. The particles observed in this event are presumably mainly protons with energies $\sim$ 1 MeV. We find that $V' = 404$ km. sec.$^{-1}$, $\mu = 4$, and we have shown the curves for both $\kappa = 1 \times 10^{18}$ cm.^{2}sec.$^{-1}$ and $\kappa = 2 \times 10^{18}$cm.^{2}sec.$^{-1}$. The solar wind speed was assumed to be 300 km.sec^{-1} in agreement with the observed wind speed ahead of the shock, and the shock

speed was taken to be 600 km. sec.$^{-1}$. The particles observed at 1 A.U. were assumed to be accelerated only over the distance 0.8 - 1.0 A.U. These curves provide reasonable fits to the observed intensities but clearly they do not predict the structure which is observed in the peaks.

III. CONCLUDING REMARKS

It should be noted that the diffusion coefficient (κ) used in the above discussion is essentially the diffusion coefficient in a direction parallel to the shock normal, not the diffusion coefficient along the field lines. Even though the particles undergo extensive scattering, they should propagate primarily along field lines, which can be at a large angle with the shock normal. Should this be the case, the diffusion coefficient parallel to the shock normal (κ_n) is then much smaller than the diffusion coefficient along the field ($\kappa_{\parallel}$) since $\kappa_n \approx \kappa_{\parallel} \cos^2\chi$, where χ is the angle between the normal and the field (Axford, 1965). Indeed, a distinguishing feature between shocks which have accompanying particle increases and those which do not may well be that in the former case the field is nearly alligned with the shock front, and hence κ_n is small resulting in an accumulation of particles.

The predicted pulse shapes agree reasonable well with the observations when $\kappa_n \sim 10^{18}$ cm.^{2}sec.$^{-1}$ for 1 MeV protons, which implies, on taking $\chi \sim 70°$, that the corresponding value for $\kappa_{\parallel} \sim 10^{19}$ cm.^{2}sec.$^{-1}$. Diffusion coefficients of this general magnitude, are consistent with the recent findings of Krimigis, et al. (1970), who in their study of the behavior of protons with energies $\gtrsim 0.3$ MeV, conclude that low energy particles undergo extensive scattering at least near the orbit of earth.

Finally, we note that in the larger events observed, the energy density of the particles at the shock front may be comparable with the energy density of the field (Armstrong, et al., 1970). It is at least conceivable that the particle increases will cause the field to be highly irregular near the shock front, although if such is the case it is doubtful whether the particle behavior can be treated using the simple diffusion picture considered here.

In summary, we have shown in this paper that many of the features observed in the pulse-like increases in the low energy cosmic ray intensity that occur at the front of propagating interplanetary shock waves can be accounted for by a simple model in which low energy particles are swept up by the shock, but because of extensive scattering by magnetic field irregularities remain near the shock front forming a pulse. We find that the intensity can increase substantially at the shock as particles gain energy by making repeated collisions with the moving shock front. It should be noted, however, that in obtaining the solutions used in illustrating the increases, we have assumed that the particles behave at the shock in a rather idealistic manner. Nevertheless, the solutions should indicate some of the general features expected for particle increases at shock fronts, and they do appear to provide reasonable fits to observed increases provided that the diffusion coefficient along the field lines is $\sim 10^{19}$ cm.^{2}sec.$^{-1}$ for 1 MeV protons.

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APPENDIX A

We assume that the required solution is of the form $U(r,T,t) = U_i\, u(r,T)$, where $U_i = AT^{-\mu}r^{\lambda}$ to the initial number density (see equation (5)). The Fokker-Planck equation (equation (3)) can then be written as an equation for u which, with $\kappa = \kappa_o r$, V a constant, and $\alpha = 2$, becomes

$$\frac{\partial u}{\partial t} = \kappa_o r \frac{\partial^2 u}{\partial r^2} + \kappa_o(1 - a) \frac{\partial u}{\partial r} \tag{A1}$$

where $a = V/\kappa_o - 2 - 2\lambda$.

As can be seen by inspection, equation (A1) is satisfied by a solution of the form $u = u(\eta)$ where $\eta = r/V_s t$ (V_s is the constant speed of the shock) is a similarity variable. On replacing the partial derivatives with respect to t and r with total derivatives with respect to η, equation (A1) becomes

$$\frac{\kappa_o \eta}{V_s} \frac{d^2 u}{d\eta^2} + \frac{\kappa_o}{V_s}\left((1 - a) + \eta\right)\frac{du}{d\eta} = 0 \tag{A2}$$

which is satisfied by

$$u = A' \,\Gamma(a, \eta) + B \tag{A3}$$

where $\Gamma(a,z) = \int_z^{\infty} e^{-w} w^{a-1} dw$ is the incomplete gamma function; A' and B are constants. We require that $U \rightarrow U_i$ as $r \rightarrow \infty$ and hence $B = 1$. A' (see equation (6)) is obtained by requiring that at the shock front ($\eta = 1$) the streaming determined by equation (2) equals the streaming determined by equation (4).

APPENDIX B

We assume that the required solution of equation (11) is of the form $U(r,T,t) = U_i(u(r,t) + 1)$, where $U_i = AT^{-u}$ is the initial number density. Note that $u(r,t)$ must also satisfy equation (11), which in terms of the variables t and $\zeta = r - V_s t$ (V_s is the constant speed of the shock), with V and κ constants, becomes:

$$\frac{\partial u}{\partial t} - V_1 \frac{\partial u}{\partial \zeta} = \kappa \frac{\partial^2 u}{\partial \zeta^2} \tag{B1}$$

where $V_1 = V_s - V$. Equation (B1) is to be solved subject to the condition that at $r = V_s t$ the streaming determined by equation (2) equals the streaming determined by equation (4), or equivalently that

$$V_2 Cu + \kappa \frac{\partial u}{\partial \zeta} + V_2 C = 0 \text{ at } \zeta = 0 \tag{B2}$$

where $V_2 = V_s - V'$, $C = (1 + 2\mu)/3$, and we have taken $\alpha = 2$.

A Laplace transform of equations (B1) and (B2) with respect to t yields

$$p\bar{u} - V_1 \frac{d\bar{u}}{d\zeta} = \kappa \frac{d^2 \bar{u}}{d\zeta^2} \tag{B3}$$

and

$$V_s C\bar{u} + \kappa \frac{d\bar{u}}{d\zeta} + \frac{V_2 C}{p} = 0 \text{ at } \zeta = 0 \tag{B4}$$

where $\bar{u} = \int_0^\infty e^{-pt}\, u\, dt$. On requiring that $\bar{u} \to 0$ as $r(\text{or } \zeta) \to 0$, the solution to equation (B3), subject to the condition given in (B4), is found to be

$$\bar{u} = \frac{V_2 C \exp(-\rho\zeta)}{p(V_2 C - \kappa\rho)} \tag{B5}$$

where $\rho = - V_1/2\kappa - \sqrt{[V_1^2/4\kappa + p/\kappa]}$. Inverting, we obtain (Erdelye, 1954)

$$u(\zeta, t) = \frac{V_2 C}{\sqrt{\kappa}} \exp\left(-\frac{V_1 \zeta}{2\kappa}\right) \int_0^t \exp\left(-\frac{V_1{}^2 u}{\kappa}\right) \left[\frac{\exp(-\zeta^2/4\kappa u)}{\sqrt{\pi u}}\right. \tag{B6}$$

$$\left. + \frac{(V_2 C - V_1/2)}{\sqrt{\kappa}} \exp((V_2 C - V_1/2)(V_2 C - V_1/2)u - \zeta)/\kappa)\, \mathrm{erfc}\left(\frac{\zeta}{2\sqrt{\kappa u}} - (V_2 C - V_1/2)\sqrt{\frac{u}{\kappa}}\right) du\right]$$

The integral in equation (B6) can be performed (yielding the solution for U given in equation (12)) by reducing it in a straightforward manner to terms involving integrals of the form (Abramowitz and Stegun, 1964):

$$\int_o^t \exp\,(-\gamma u - \delta/4u)\frac{du}{\sqrt{\pi u}} = \frac{1}{2\sqrt{\gamma}}\left[\exp\,(-\sqrt{\delta\gamma})\ \mathrm{erfc}\ (\tfrac{1}{2}\ \sqrt{\tfrac{\delta}{t}}\ - \sqrt{\gamma t}) - \exp\,(\sqrt{\delta\gamma})\ \mathrm{erfc}\ (\tfrac{1}{2}\ \sqrt{\tfrac{\delta}{t}}\ + \sqrt{\gamma t})\right] \tag{B7}$$

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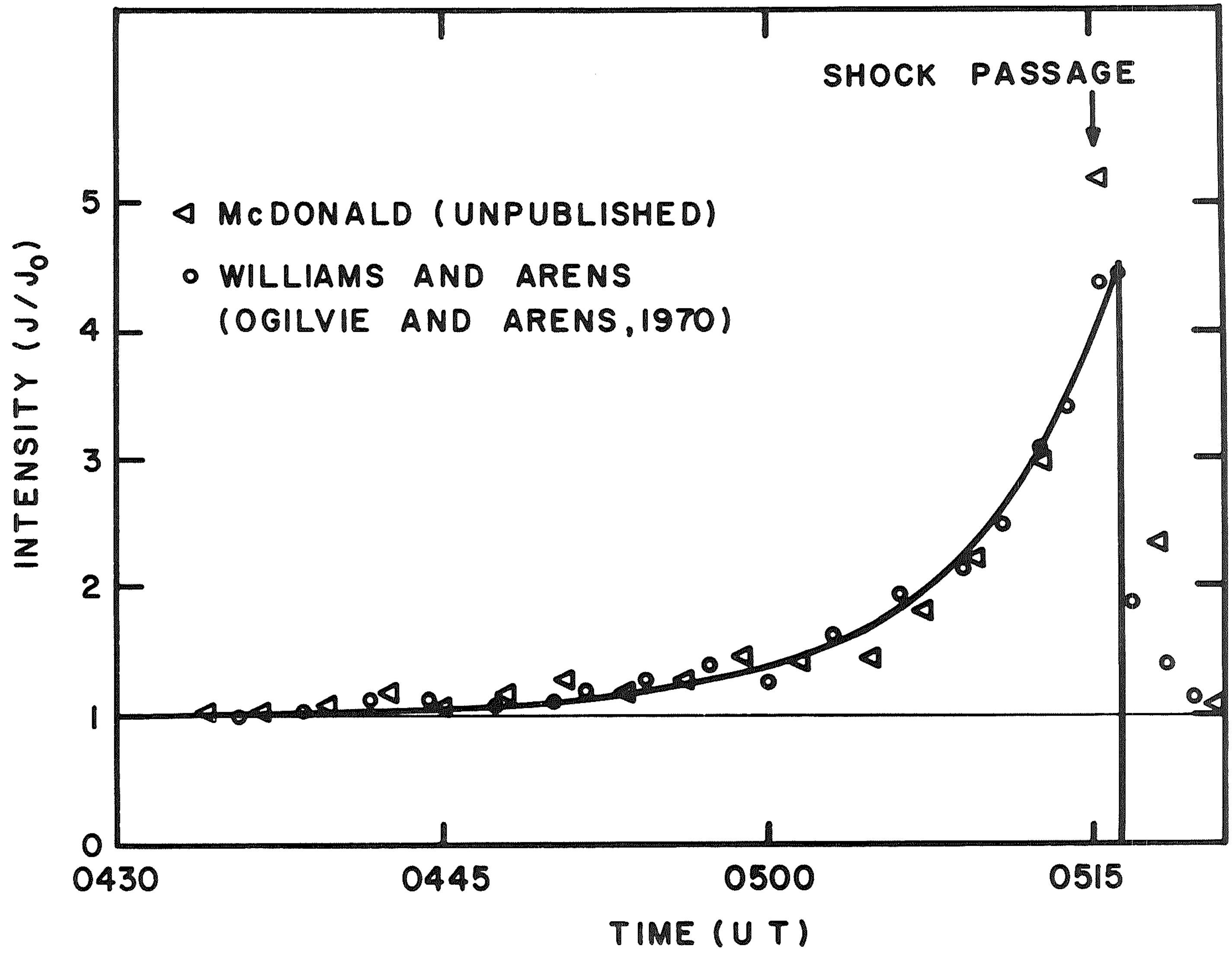
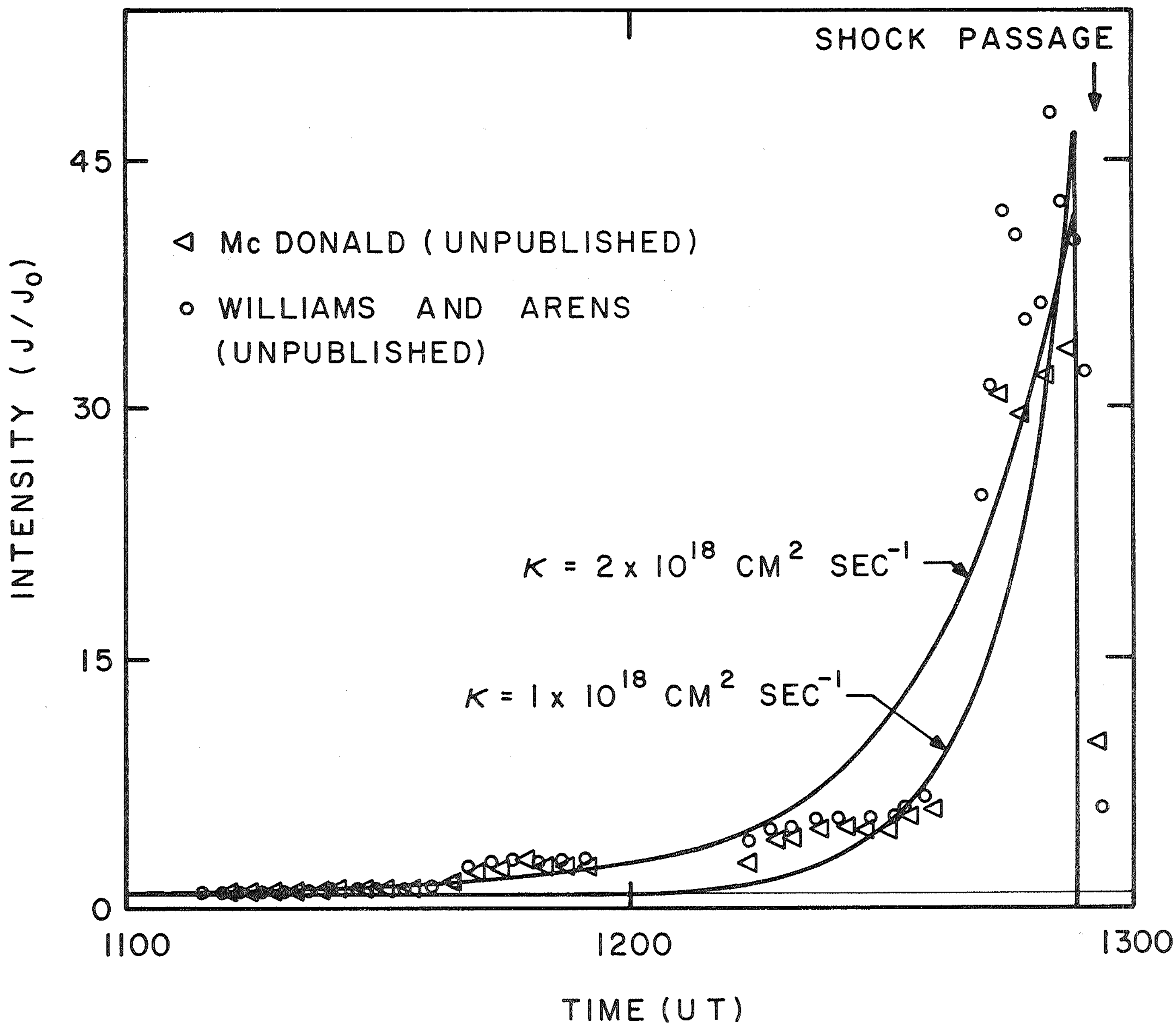
FIGURE CAPTIONS

FIGURE 1. A plot of the relative intensity increase determined by the solution given in equation (6) using the values for κ_o, V', and μ which give the best fit to the data obtained by McDonald (unpublished), and by Williams and Arens (Ogilvie and Arens, 1970) for the event observed on 29 November 1967. $J_o = vU_i/4\pi$ is the undisturbed intensity ahead of the shock. There is an uncertainty of several minutes in the location of the data points, which are determined from IMP 4 raw counting rates, and also in the time of the shock passage, which is shown here as occuring at the time reported by Ogilvie and Burlaga (1968).

FIGURE 2. A plot of the relative intensity increase determined by the solution given in equation (12) using the values for κ, V', and μ which give the best fit to the data obtained by McDonald (unpublished), and by Williams and Arens (unpublished) for the event observed on 11 January 1968. $J_o = vU_i/4\pi$ is the undisturbed intensity ahead of the shock. There is an uncertainty of several minutes in the location of the data points, which are determined from IMP 4 raw counting rates, and also in the time of the shock passage, which is shown here as occuring at the time reported by Ogilvie and Burlaga (1968). It should be noted that in this 11 January event the intensity increased again following its abrupt drop at the shock passage. We have assumed here that this increase following the shock is unrelated to the mechanism discussed here.



SHOCK PASSAGE
McDONALD (UNPUBLISHED)
WILLIAMS AND ARENS
(OGILVIE AND ARENS, 1970)
INTENSITY (J/J_0)
0
1
2
3
4
5
0430
0445
0500
0515
TIME (U T)


SHOCK PASSAGE
45
30
15
0
INTENSITY (J / J0)
Mc DONALD (UNPUBLISHED)
WILLIAMS AND ARENS (UNPUBLISHED)
K = 2 x 10^18 CM^2 SEC^-1
K = 1 x 10^18 CM^2 SEC^-1
1100
1200
1300
TIME (U T)